Irregular Labelings of Circulant Graphs

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We investigate the irregularity strength ($s(G)$) and total vertex irregularity strength ($tvs(G)$) of circulant graphs $Ci_n(1, 2, \ldots, k)$.

The values of $s(G)$ and $tvs(G)$ have been studied by numerous authors. The best known upper bounds for the general case are $s(G) \leq \lceil \frac{6n}{\delta} \rceil$ ([3]) and $tvs(G) \leq \lceil \frac{3n}{\delta} \rceil + 1$ ([1]). The exact values for some families of graphs are also known, e.g. the value of $s(Ci_n(1, k))$, given in [2].

We prove that $tvs(Ci_n(1, 2, \ldots, k)) = \frac{n+2k}{2k+1}$, while $s(Ci_n(1, 2, \ldots, k)) = \frac{n+2k-1}{2k}$. In order to do that, we split the graph $Ci_n(1, 2, \ldots, k)$ into segments and label each segment using 0, 1 and 2 in such a way that the weighted degrees of the vertices included in that segment are distinct multiplicities of 2. In the next step we multiply all the edge labels by about $s/2$ (depending on the parity of $s$) in order to obtain the labeling where all the weighted degrees in any chosen segment differ by at least $s$. Then by changing the weighted degrees in every segment by distinct integer from the set \{1, 2, \ldots, s\} we obtain the desired irregular weighting.

References

